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ANALYSIS OF PROTON SCATTERING FROM CARBON ISOTOPES

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Abstract

Reaction cross section of carbon isotopes for proton scattering is computed with the wide incident energy region. Calculations are based on eikonal approximation which is a high energy (small scattering angle) that depends on the nucleon nucleus (NA) optical potential. The proton-carbon optical potential is obtained by a folding integral of the nucleon nucleon (NN) transition amplitude and matter density of carbon. Analytical formulas of optical potential are used to present in position-space representation for carbon isotope by using harmonic well nuclear densities(A<20). The eikonal phase function is presented in the momentum-space representation which is obtained from computing the fourier transform of the position-space optical potential in order to compare each other.

Keywords: Eikonal approximation, Optical potential, Reaction cross section

Introduction

In this present work, proton-carbon isotope scattering will be studied in order to predict the size of carbon isotope in the frame work of eikonal approximation. The scattering of particles from nuclei has provided invaluable information on charge, matter densities distribution and root-mean-square radius of stable nuclei near the stability line. Proton elastic scattering is expected to be the most suitable experiment besides electron scattering in order to obtain such information on the stable nuclei. For scattering problem of composite particle, the exact solution of Schrodinger is very difficult. So the appropriate approximation is indispensable. In this study, the eikonal approximation is used to calculate the differential cross section and reaction cross section of proton carbon scattering. It is well-suited for the prediction of cross sections for projectile with kinetic energies in the laboratory frame greater than 150 MeV/n. The differential cross section is computed from the absolute square of scattering amplitude that is obtained by integrating the eikonal phase function. The eikonal phase function is related to the optical potential which depends on NN transition amplitude and density profile of carbon. In this study, nuclei are composite particles whose fundamental constituents are nucleons (protons and neutrons). The quark structure of the nucleons is not considered. It is assumed that the inner structure of nucleon will be probed at higher energies, and these effects are considered to be included in the NN transition amplitude. The NN transition amplitude is a function of total cross section, slope parameter and real to imaginary ratio. Electron scattering experiments are used to estimate the charge density of nuclei. Harmonic well densities are used for light nuclei. Nuclear matter density is obtained from dividing the nuclear charge density by the Gausian charge distribution of the proton. In the position-space representation, the optical potential is a six dimensional nuclear matter densities and the NN transition amplitude. In momentum space representation, it is the product of the matter densities and NN transition amplitudes as a function of momentum transfer.

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Theory

Eikonal Approximation

The time independent Schrodinger equation is given by

$$\hat{H}|\psi>=E|\psi> \qquad (1)$$

$$\hat{H}=\hat{H}_0+\hat{V}$$

$\hat{H}$ = Halmiltonian, $\hat{V}$ =potential energy operator and $\hat{H}_0$ = kinetic energy operator

$$(\hat{H}_0+\hat{V})|\psi>=E|\psi>$$

$$|\psi>=\frac{V}{E-\hat{H}_0}|\psi> \qquad (2)$$

To avoid the singular nature of the operator $\frac{1}{E-\hat{H}_0}$, $\pm i\varepsilon$ is added into equation (2)

$$|\psi^{\pm}>=\frac{V}{E-\hat{H}_0\pm i\varepsilon}|\psi^{\pm}> \qquad (3)$$

The solution of above equation is given by

$$|\psi^{\pm}>=|\phi>+\frac{V}{E-\hat{H}_0\pm i\varepsilon}|\psi^{\pm}> \qquad (4)$$

This is known as Lippmann-Schwinger equation

Where, $|\phi>$ is the solution of free particle Schrödinger equation, $\psi^{\pm}$ is

outgoing (incoming) wave function

By using the completeness relation, we obtained

$$<\vec{r}\,|\psi^{\pm}>=<\vec{r}\,|\varphi>+\int d\vec{r}'<\vec{r}\,|\frac{1}{E-\hat{H}_0\pm i\varepsilon}|\vec{r}'><\vec{r}'|V||\psi^{\pm}> \qquad (5)$$

Eikonal approximation is useful approximation techniques when the de Broglie wavelength λ = h/p of the incident particle is sufficiently short compared with the distance in which the potential varies appreciably. If the potential varies smoothly and has a range "a", this short wavelength condition is equivalent to the requirement that ka >>1.

Let's consider high energy, non-relativistic potential and assume that ka >> 1 (short wavelength) and $V_0/E<<1$ (high energy)

We start the Lippmann Schwinger equation

$$\Psi_{\vec{k}i}^{(+)}(\vec{r}) = \frac{e^{(i\vec{k}_i \bullet \vec{r})}}{(2\pi)^{3/2}} + \int G_0^{(+)}(\vec{r},\vec{r}\,')\ \Psi_{\vec{k}i}^{(+)}(\vec{r}')\ U(\vec{r}\,')d\vec{r}\,' \tag{6}$$

Where, $U(\vec{r}) = \dfrac{2mV(r)}{\hbar^2}$, strength of reduce potential

Where, the Green's function is given by

$$G_0^{(+)}(\vec{r},\vec{r}\,') = -\frac{1}{4\pi}\frac{e^{\{ik|\vec{r}-\vec{r}'|\}}}{|\vec{r}-\vec{r}\,'|}$$

$$G_0^{(+)}(\vec{r},\vec{r}') = -\frac{1}{(2\pi)^3}\int d\vec{\kappa}\,\frac{e^{\{i\vec{k}\bullet(\vec{r}-\vec{r}')\}}}{\kappa^2 - k_i^2 - i\varepsilon} \tag{7}$$

When the potential varies slowly over the scale of the incident wavelength, the full wave function Ψ_k^+, is given by

$$\Psi_{ki}^{(+)}(\vec{r}) = \frac{e^{i\vec{k}_i \bullet \vec{r}}}{(2\pi)^{3/2}}\phi(\vec{r}) \tag{8}$$

Where, $\phi(\vec{r})$ is slowly varying function when ka is large.

The eikonal scattering wave function is obtained by

$$\Psi_E^{(+)}(\vec{r}) = \frac{1}{(2\pi)^{3/2}}\exp(i\vec{k}_i \bullet \vec{r} - \frac{i}{2.k}\int_{-\infty}^{z} U(x,y,z')\,dz') \tag{9}$$

Calculation of eikonal scattering amplitude

Eikonal scattering wave function is given by

$$\Psi_E^{+}(\vec{r}) = \frac{1}{(2\pi)^{3/2}}\exp\{i\vec{k}_i\vec{r} - \frac{i}{2k}\int_{-\infty}^{Z} U(x,y,z')\,dz'\}$$

The scattering amplitude and the scattered wave function is related by

$$f_E = -\frac{(2\pi)^{3/2}}{4\pi}\int e^{(-i\vec{k}_f\vec{r})}U(\vec{r})\psi_E^+(\vec{r})d\vec{r} \tag{10}$$

Substituting $\psi_E^+(\vec{r})$ into (10)

$$f_E = -\frac{(2\pi)^{3/2}}{4\pi}\int e^{(-i\vec{k}_f\vec{r})}U(\vec{r})\frac{1}{(2\pi)^{3/2}}\exp\{i\vec{k}_i\vec{r} - \frac{i}{2k}\int_{-\infty}^{Z} U(x,y,z')\,dz'\}\,d\vec{r}$$

$$f_E = -\frac{1}{4\pi}\int d\vec{r}e^{i(\Delta.\vec{r})}U(\vec{r})\exp\{-\frac{i}{2k}\int_{-\infty}^{Z} U(x,y,z')\,dz'\}$$

Where, $\Delta = \vec{k}_i - \vec{k}_f$ is wave vector transfer

For cylindrical coordinate system, $d\vec{r} = d^2\vec{b}d\vec{z}$

$$f_E = -\frac{1}{4\pi}\int d^2\vec{b}\int_{-\infty}^{\infty} d\vec{z}e^{i(\Delta.\vec{b})}U(\vec{b},z)\exp\{-\frac{i}{2k}\int_{-\infty}^{Z} U(\vec{b},z')dz'\} \quad (11)$$

z' integration in (11) along a $\hat{n}$ parallel to the bisector of the scattering angle .

$\vec{r} = \vec{b} + z\vec{k}_i$ and $\Delta.\vec{r} = \Delta(\vec{b} + z\vec{k}_i) = \Delta.\vec{b} + kz(1-\cos\theta) \approx \Delta.\vec{b}$

Now eikonal scattering amplitude becomes as below

$$f_E = \frac{k}{2\pi.i}\int d^2\vec{b}e^{i(\Delta\bullet\vec{b})}[\exp\{i\chi(k,b)\}-1] \quad (12)$$

Where, eikonal phase shift function is $\chi(k,\vec{b}) = -\frac{1}{2k}\int_{-\infty}^{+\infty} U(\vec{b},z)dz$ (13)

Where, $d^2\vec{b} = bdbd\phi$ for cylindrical coordinate system

So, $$f_E = \frac{k}{2\pi i}\int_0^{\infty}\int_0^{2\pi} bdbd\varphi e^{i(\Delta.\vec{b})}[\exp\{i\chi(k,b)\}-1]$$

$$f_E = \frac{k}{2\pi i}\int_0^{\infty} bdb\int_0^{2\pi} d\varphi e^{i(\Delta.b\cos(\varphi))}[\exp\{i\chi(k,b)\}-1]$$

The ordinary Bessel Function J_0 is given by

$$J_0(x) = \frac{1}{2\pi}\int_0^{2\pi} d\varphi e^{(ix\cos\varphi)} \quad (14)$$

$$f_E = \frac{k}{i}\int_0^{\infty} bdbJ_0(\Delta.b)[\exp\{i\chi(k,b)\}-1] \quad (15)$$

where, b is the impact parameter, k is the relative momentum of the projectile-target system in the center of mass (CM) frame.

$$k = \sqrt{\frac{2\mu m_{At}T_{lab}}{m_{A_p} + m_{A_T}}}$$

where, $\mu = \frac{m_{A_p}m_{A_T}}{m_{A_p} + m_{A_T}}$, T_{lab} is the kinetic energy of the projectile in laboratory frame.

The elastic differential cross section is computed from the absolute square of the scattering amplitude.

$$\frac{d\sigma}{d\Omega} = \left| f(\theta) \right|^2 \tag{16}$$

Total elastic cross section is obtained by integrating over the solid angle

$$\sigma_{\mathrm{el}}^{tot} = \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\mathrm{d}\Omega$$
$$= 4\pi \int_0^\infty [1 - e^{-\operatorname{Im}\chi}\cos(\operatorname{Re}\chi)]bdb - 2\pi\int_0^\infty [1 - e^{-2\operatorname{Im}\chi}]bdb \tag{17}$$

The scattering amplitude is satisfies the optical theorem, so the total cross section is given by

$$\sigma_{tot} = \frac{4\pi}{k}\operatorname{Im} f(\theta = 0) = 4\pi\int_0^\infty [1 - e^{-\operatorname{Im}\chi}\cos(\operatorname{Re}\chi)]bdb \tag{18}$$

The reaction cross section is obtained by $\sigma_{\mathrm{re}} = \sigma_{tot} - \sigma_{el}$

$$\sigma_{\mathrm{re}} = 2\pi\int_0^\infty [1 - e^{-2\operatorname{Im}\chi}]bdb \tag{19}$$

Optical Potential

Differential cross sections are function of eikonal phase function which depends on the optical potential, as shown in equation (13). For Nucleus-Nucleus (AA) scattering, the optical potential can be expressed as [9]

$$U(r) = A_p A_T \int t_{NN}(|\vec{r}_{NN}|)\rho_p(|\vec{r}_P|)\rho_T(|\vec{r}_T|)d\vec{r}_T d\vec{r}_{NN} \tag{20}$$

Where, A is the number of nucleon, p is projectile, T represent the target, t_{NN} is NN transition amplitude and ρ is nuclear matter density. The vectors used for the AA optical potential are illustrated in figure (1). The distance from the center of the projectile nucleus to a nucleon in the projectile can be expressed as $\mathbf{r}_{\mathrm{p}} = \mathbf{r} + \mathbf{R} = \mathbf{r} + \mathbf{r}_{\mathrm{T}} + \mathbf{r}_{\mathrm{NN}}$, which , when substituted into (20),

$$U(r) = A_p A_T \int t_{NN}(|\vec{r}_{NN}|)\rho_p(|\vec{r} + \vec{r}_T + \vec{r}_{NN}|)\rho_T(|\vec{r}_T|)d\vec{r}_T d\vec{r}_{NN} \tag{21}$$

Where, $|\vec{r}| = \sqrt{b^2 + z^2}$ in the cylindrical coordinate system.

The eikonal phase function can be obtained by integrating the optical potential in the position-space representation. In this section, it is expressed as a function of momentum transfer in order to reduce the number of integration dimensions.

Nucleon-Nucleon Transition Amplitude and Nuclear Matter Density

In present work, differential cross section is calculated with the eikonal approximation using the momentum-space representation of the optical potential, which depends on the nucleon-nucleon (NN) transition amplitude and nuclear matter density. The harmonic-well nuclear matters density is given by [9]

$$\rho_A(r) = \frac{\rho_0 a^3}{8s^3}[(1 + \frac{3\gamma}{2} - \frac{3\gamma a^2}{8s^2}) + \frac{\gamma a^2}{16s^4}\gamma^2]e^{(\frac{-r^2}{4s^2})} \tag{22}$$

$$\rho_0 = \frac{1}{\pi^{3/2} a^3 [1+\frac{3}{2}\gamma]} \qquad s = \sqrt{(\frac{a^2}{4} - \frac{r_p^{\ 2}}{6}}$$

The Fourier transform of the harmonic-well nuclear matters density is given by

$$\rho_A(q) = \rho_0 \pi^{3/2} a^3 [(1+\frac{3\gamma}{2} - \frac{3\gamma a^2}{4} q^2)] e^{(q^{-2} s^2)} \tag{23}$$

The simplest form of NN transition amplitude is expressed as follow

$$t_{NN}(r) = -\sqrt{\frac{e}{m_p}} \frac{\hbar}{[2\pi B(e)](3/2)} \sigma(e)[\kappa(e)+i] e^{-r^2/2B(e)} \tag{24}$$

where, e is the kinetic energy of two nucleon center of momentum system, B(e) is the slope parameter of pp(pn) elastic scattering cross section, σ(e) is the pp(pn) cross section and $\kappa(e)$ is the real to imaginary ratio of the pp(nn) cross section. The input parameters of NN transition amplitude are displayed in table (1). The Fourier transform of NN Transition amplitude is given by

$$t_{NN}(q) = \frac{-\hbar^2 k \sigma(e)}{16\pi^3 \mu} [\kappa(e) + i] e^{-\frac{B(e) q^2}{2}} \tag{25}$$

where $\hbar$ is Planck's constant, μ is the reduced mass of the NN system and k is the relative momentum in the CM frame.

For the case of Nucloen-Nucleus (N-A) collision, optical Potential in position space is computed by using harmonic nuclear matter density and NN transition amplitude.

$$U(r) = (C_0 + C_1 r^2) e(-C_2 r^2) \tag{26}$$

where

$$C_0 = \tau A_T (\frac{\pi}{\mu_1 + \mu_2})^{3/2} [\alpha_T + \frac{3\beta_T}{2(\mu_1+\mu_2)}] \qquad C_1 = \frac{\tau \alpha_T \beta_T \mu_1 \pi^{3/2}}{(\mu_1+\mu_2)^{7/2}} \qquad C2 = \mu_1 + \frac{\mu_1^{\ 2}}{(\mu_1+\mu_2)}$$

$$\tau_{NN}(r) = -\sqrt{\frac{e}{m_p}} \frac{\hbar}{[2\pi B(e)](3/2)} \sigma(e)[\kappa(e)+i]$$

$$\mu_1 = \frac{1}{2B} \qquad \mu_2 = \frac{1}{4 s_T^2} \qquad s = \sqrt{(\frac{a^3}{4} - \frac{r_p^{\ 2}}{6})}$$

Optical Potential in momentum space is obtained with the help of fourier transforms of the NN transition amplitude and nuclear matter density.

$$U(q) = A_p A_T t_{NN}(\vec{q}) \rho_p(\vec{q}) \rho_T(\vec{q}) \tag{27}$$

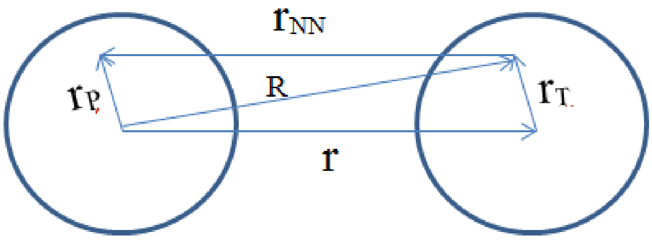


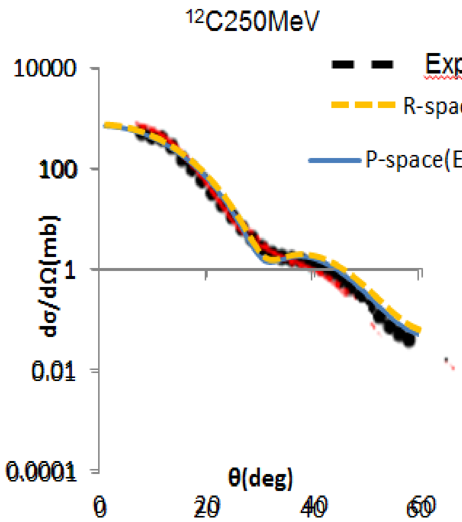
Figure 1. Illustration of vectors used for the AA optical potential. r_{NN} is the vector between a nucleon in the projectile and a nucleon in the target; $\mathbf{r}_p$ is the vector that extends from the center of projectile to a nucleon in the projectile; $\mathbf{r}_T$ is the vector between the center of the target nucleus to a nucleon in the target; **r** is the relative distance between the centers of the projectile and target nuclei; $\mathbf{R} = \mathbf{r} + \mathbf{r}_T$ is the from the center of projectile to a nucleon in the target B.

Table 1 Parameter of NN transition amplitude

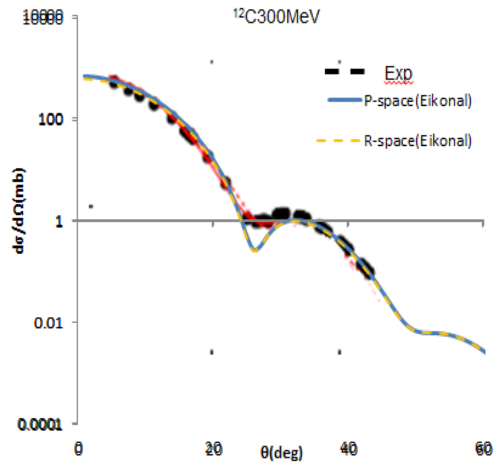
E(MeV)	**σ(mb)**	**κ**	**B (fm^2)**
30	19.6	0.87	0.685
40	14.4	0.9105	0.462
50	10.4	0.94	0.390
60	9.15	1.173	0.376
70	8.01	1.27	0.354
80	6.79	1.324	0.326
100	5.51	1.37	0.281
160	4.17	1.183	0.173
200	3.405	0.961	0.126
300	3.06	0.476	0.074
425	3.01	0.36	0.741
550	3.47	0.04	0.098
650	3.94	-0.19	0.13
800	4.255	-0.07	0.153
1000	4.52	-00272	0.172

Result and Discussion

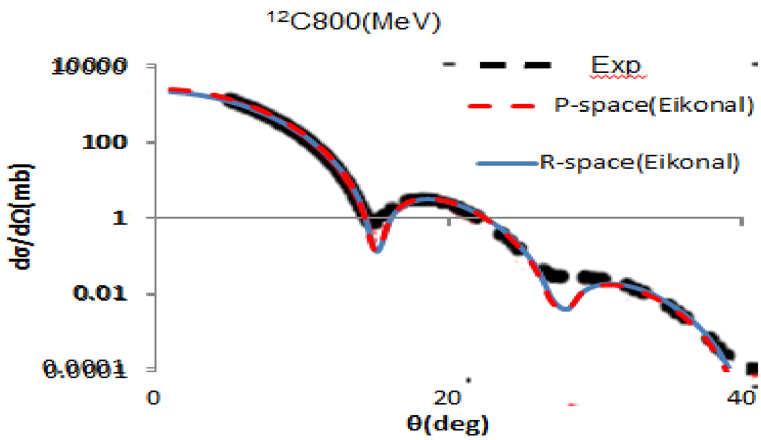
The differential cross section for proton- carbon isotopes scattering ($^{12,13,14}C$) have been calculated by using optical potential. The optical potential depends on parameterization of the harmonic well nuclear matter density and NN transition amplitude. The harmonic well nuclear density parameters used in equation (22)-(23) for carbon isotopes are a=1.67 ϒ= 1.607 (^{12}C), a=1.64 ϒ= 1.432 (^{13}C) and a=1.671 ϒ= 1.26 (^{14}C). The energy dependent parameters of NN transition amplitude are displayed in table 1. In figure 2,3 and 4, the eikonal (position space) refers to the numerical evaluation of the optical potential equation (20), via Gaussian Quadrature Method (six-dimensional integral). Equations (13) and (15) were integrated numerically using Gaussian Quadrature in order to obtain the differential cross section for proton- carbon Isotope. But in eikonal (momentum space) the numerical calculation only required phase shift function equation (13) (one-dimensional integral). So the momentum space eikonal phase shift function was then used directly in equation (15). The calculated differential cross section of carbon isotopes which is various incident energies are shown in figure 2 to 4. That figures show the differential cross section predicted with position space calculation and momentum space calculation agree with experimental data. And also note that position space and momentum space results are good agreement with each other. Comparison of reaction cross section between results which is obtained from the theoretical and experimental calculation is shown in figure 5 in the case of proton carbon scattering. The red dot line represents the currently eikonal calculation. The solid yellow line and solid green line are results from other theoretical calculations of and. The solid black circles are experimental data taken from. It was found that the calculated reaction cross section for p-carbon ^{12}C reaction in eikonal approximation agrees with experimental data as well as other theoretical calculations. Figure 6 shows the reaction cross section for p- ^{13,14}C reaction with respect to energy. According to figure 5 and 6, the reaction cross section of proton carbon isotope increases with increasing mass number of carbon isotope.



(a) Result for 250MeV

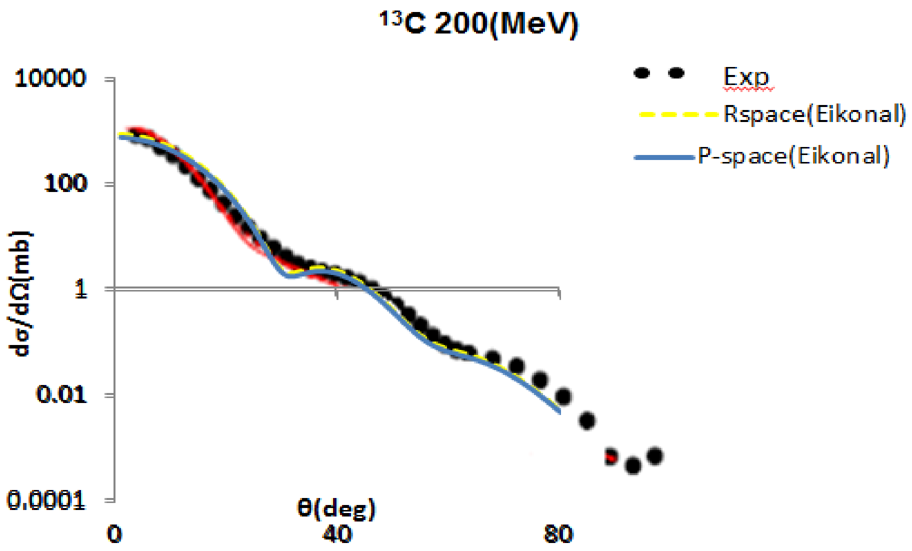


(b) Result for 300MeV

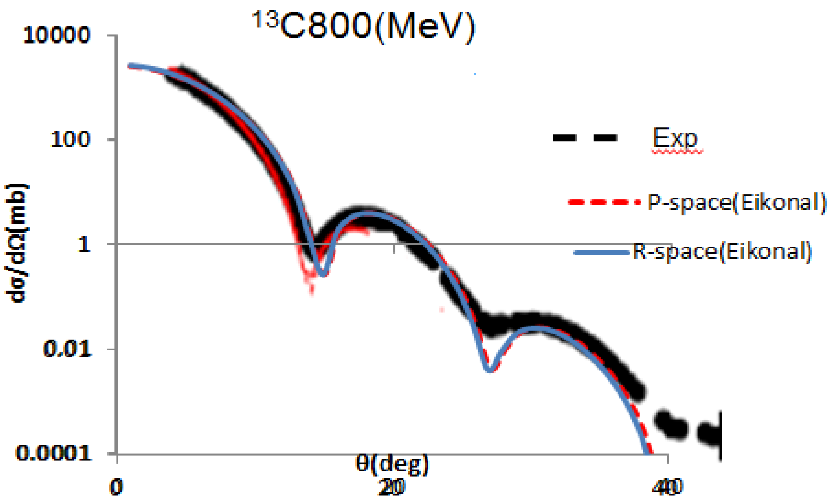


(c) Result for 800MeV

Figure 2 Differential cross section for Carbon ^{12}C target incident on proton at (a) 250MeV, (b) 300Mev and (c) 800MeV. Color lines are p-space and R-space eikonal calculation. Experimental data



(d)Result for 200MeV



(e) Result for 800MeV

Figure 3 Differential cross section for p-carbon ^{13}C scattering at (d) 200MeV and (e) 800MeV. Experimental data are taken from Ref [4].

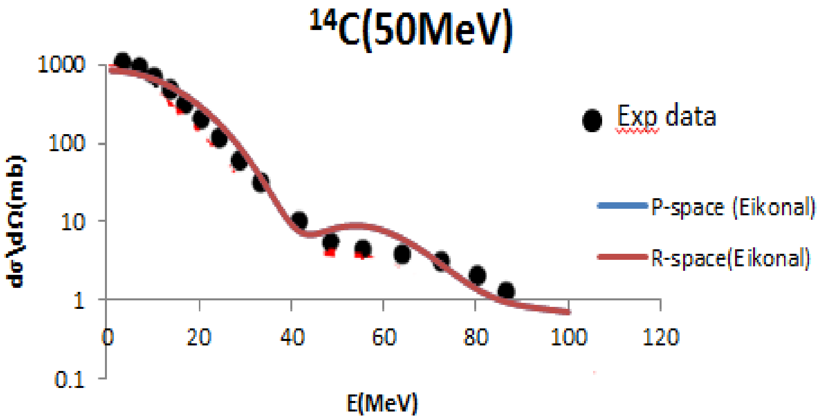


Figure 4 Comparison of numerical results (p-space and R-space) with experimental data [4] for P-Carbon ^{14}C scattering

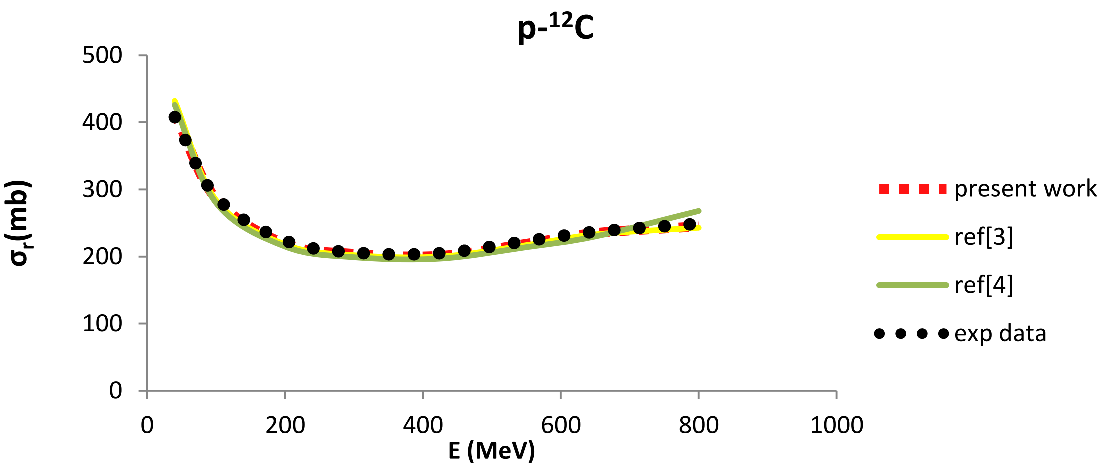
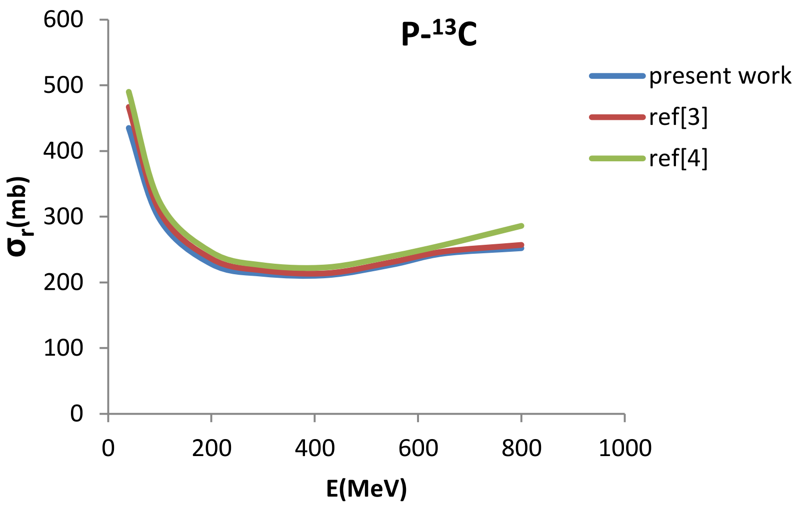


Figure 5 Comparison study of reaction cross section of P-Carbon ^{12}C scattering as a function of energy



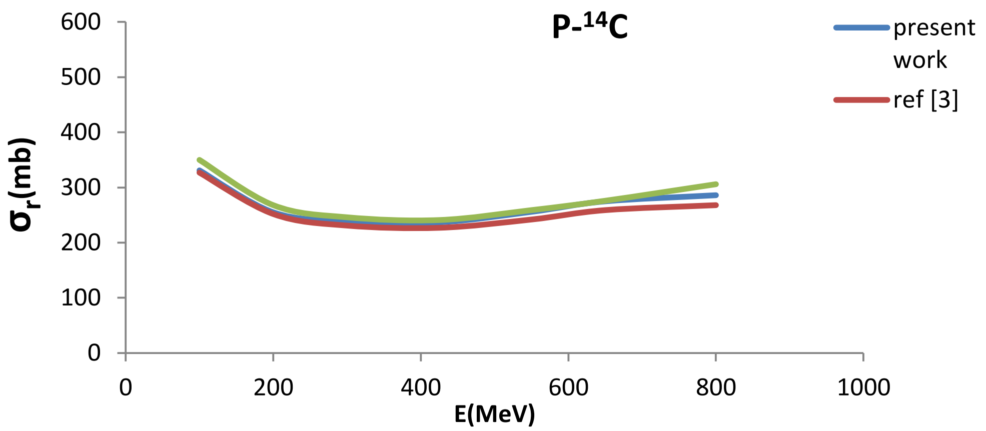


Figure 6 Comparison of reaction cross section for P- Carbon ^{13}C (upper) & ^{14}C(lower) scattering between present work and other theoretical results

Conclusion

The differential cross section of proton carbon scattering is predicted by using NN optical potential. It is obtained by computing a six- dimensional integral over the nuclear matter densities of the target and NN transition amplitude. Consequently, numerical calculation of optical potential is inefficient. According to numerical calculation, the eikonal phase function can be written as a one-dimensional integral by expressing the optical potential as a function of momentum transfer, thereby greatly increasing the efficiency of the numerical evaluation of cross section using eikonal approximation. Moreover, NA optical potential were obtained with harmonic well nuclear matter densities, which are suitable for light nuclei (A<20). The formulas were used to predict the elastic differential cross sections for proton-light nuclei reactions. The results generated from the optical potential were verified with numerical integration, and it was fond that elastic differential cross section are in good agreement with experimental data displayed in figure 2,3 and 4. The momentum-space formulation of the eikonal phase function is used to evaluate the differentisl cross section of proton carbon reaction which utilizes target nuclear matter density parameterization. It was found that the momentum-space phase function agrees exactly with the eikonal approximation computed in position-space and the results of both calculations are in good agreement with experimental data. In present work, the reaction cross sections for proton-elastic scattering from carbon isotopes of A=12-14 calculated in large energy region of 100-1000MeV. The results obtained from eikonal approximation with optical potential compares with other theoretical results as well as experimental results. Their behaviors of reaction cross section with respect to energy are very similar to each other. In figure 5 and 6, the eikonal approximation gives significantly large reaction cross section in energies less than 200MeV, showing minimum values at around 300-425 and after that slightly increasing with increasing energies. This effect is due to attribution of NN transition of amplitudes. The bigger reaction cross section of proton carbon isotope, the more increase mass number of isotope is due to the more present of nucleon inside the nucleus and the interaction between projectile and target becomes snowballing.

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